Algebra and Calculus of the Resource Space Model

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Abstract

Resource Space Model (RSM) is a semantic model to manage resources in the future interconnection environment. The query capability is an important aspect of RSM as a semantic resource management model. This paper reports the research result on the query capability of RSM from two perspectives: resource space algebra and resource space calculus. The equivalence of the resource space algebra and the resource space calculus has been discussed.

1. Introduction

The Resource Space Model (RSM) is a semantic data model for uniformly, normally and effectively specifying and managing heterogeneous, distributed and ocean resources in an open and dynamic Internet environment [3]. To know the query capability and make use of the potential expressive power of RSM are essential issues. The query capability and expressive power of RSM can be studied from two perspectives: resource space algebra and resource space calculus.

The resource space algebra consists of a set of resource spaces and a set of operations of the Resource Space Model. Users can use a series of operations in the resource space algebra to obtain the desired resources. To lay the foundation of the query language for Resource Space Model, we also propose a resource space calculus. The resource space calculus is a type of applied predicate calculus and a foundation for the declarative query language. By either algebra or calculus of RSM, users can easily and clearly specify the desired resources.

2. Resource Space Algebra

The resource space algebra consists of a set of resource spaces and a set of operations of RSM. Result of operations on resource spaces are also resource spaces.

Join, Disjoin, Merge and Split operations have been defined in [1, 3]. We only introduce some new operations in the algebra of RSM.

Union. For two resource spaces RS_1 and RS_2 having the same schema, the union of RS_1 and RS_2 is defined as RS such that RS has the same schema as RS_1 and the point in RS is the union of corresponding points in RS_1 and RS_2. RS is denoted as $RS_1 \cup RS_2$.

Difference. For two resource spaces RS_1 and RS_2 having the same schema, the difference of RS_1 and RS_2 is defined as RS such that RS has the same schema as RS_1 and the point in RS is the difference of corresponding points in RS_1 and RS_2. RS is denoted as $RS_1 - RS_2$.

The *intersection* operation on RS_1 and RS_2 can be defined as $RS_1 \cap RS_2 = RS_1 - (RS_1 - RS_2)$.

Cartesian product. Let $RS_1(X_1, X_2 \ldots X_n)$ and $RS_2(Y_1, Y_2 \ldots Y_m)$ be two resource spaces. The Cartesian product of RS_1 and RS_2 is defined as $RS_1 \times RS_2 = RS(X_1, X_2 \ldots X_n, Y_1, Y_2 \ldots Y_m)$.

The *projection* operation has almost the same definition as *disjoin* operation except that projection results in only one resource space which includes all the desirable axes. $\pi_{X1, \ldots, Xm}(RS)$ will be used to denote the projection of resource space RS on axes $X_1, \ldots, X_m$.

Selection. For a resource space RS, the *Selection* operation is denoted as $\sigma_F(RS)=\{p \mid p \in RS \wedge F(p)\}$, where F is a logic expression. All points in RS making F true will be selected. F has the form of $p_m[X_i]\ \theta\ Y$, where Y may be a noun and noun phrase in domain ontology and θ represents =, ≠, <, ≤, ≥ or >.

Division. $RS_1(A, B)[\div B]RS_2(B, C) = \pi_A(RS_1) - (\pi_A(RS_1) \cdot \pi_A(\pi_A(RS_1) \times \pi_B(RS_2) - \pi_A(RS_1) \times \pi_B(RS_2) \cdot RS_1))$, herein $A=X_1, \ldots, X_m$, $B=Y_1, \ldots, Y_t$ and $C=Z_1, \ldots, Z_n$.

Theorem 1 Union, Difference, Intersection, Cartesian product, Projection and Selection keep 1NF, 2NF and 3NF of the Resource Space Model.

3. Resource Space Calculus

The resource space calculus consists of *variables*, *terms*, *formulas* and *alpha expressions*.

The set ***V*** of *variables* is the countable sets {p, p_1, p_2, p_3 ...}, where each p_i stands for a point variable.

The set ***T*** of *terms* is composed of the following three parts. (1)Any nouns and noun phrases in ontology are in ***T***. (2) Any axis, the split of an axis or the merge of two axes belong to ***T***. (3) For any point variable p_i and its any axis X_j, $p_i[X_j]$ is a term.

The set ***RF*** of *range formulas* is defined as follows.

(1) Let RS_i be a resource space and point variable $p \in \boldsymbol{V}$, then $RS_i(p)$ belongs to ***RF***. (The monadic predicate $RS_i(p)$ is used to state that the point variable p has the range of resource space RS_i.)

(2) If point variable p is the only point variable in a range formula, then this range formula is called a *range formula over p*. Let Δ, Γ be two range formulas over p. Then $\Delta\wedge\neg\Gamma$ belongs to ***RF***.

(3) Let $\Delta, \Gamma \in \boldsymbol{RF}$. For any point variable p in Δ and Γ, if all resource spaces specifying the range of p in Δ and Γ are union-compatible, then both the disjunction $\Delta\vee\Gamma$ and the conjunction $\Delta\wedge\Gamma$ are in ***RF***.

The set ***F*** of *formulas* includes the following four types of formulas. (1) Any range formula in ***RF*** is in ***F***. (2) *Coordinate formula*: $p_m[X_i]\ \theta\ Y$, where Y may be $p_n[X_j]$ or just a noun and noun phrase in ***T*** and θ represents any of the relations =, ≠, <, ≤, ≥ and >. (3) If $\Delta, \Gamma \in \boldsymbol{F}$, then the negation $\neg\Delta$, the disjunction $\Delta\vee\Gamma$ and the conjunction $\Delta\wedge\Gamma$ are in ***F***. And, (4) Let Φ be a range formula over p. Then the quantification $(\exists\Phi)\Delta$ and $(\forall\Phi)\Delta$ are in ***F***.

It is obvious that in ***F*** each qualifier (∃ or ∀) must be associated with a range formula over a point variable. The expanded forms of $(\exists\Phi)\Delta$ and $(\forall\Phi)\Delta$ are as follows.

$$(\exists\Phi)\Delta = \exists p(\Phi \wedge \Delta)$$
$$(\forall\Phi)\Delta = \forall p(\neg\Phi \vee \Delta)$$

For any formula Γ in ***F***, Γ is a *well-formed formula* (WFF in simple) *over p* if Γ has the form of $U_1 \wedge ... \wedge U_n \wedge V$, where

(1) U_1 through U_n are range formulas over n point variables varying from one another;

(2) V belongs to ***F*** and satisfies:

   a) The range of every free variable except p in V has been specified by a certain U_i;

   b) No rang formula occurs in V.

Then this WFF over p is denoted as $\Gamma(p)$.

Let $\Gamma(p)$ be a WFF formula over p and X_i ($1 \le i \le n$) be a group of axes. The *alpha expression* can be defined as follows:

(1) $p(X_1, X_2, ..., X_n)$: $\Gamma(p)$ is an alpha expression;

(2) If both $p(X_1, X_2, ..., X_n)$: Δ_1 and $p(X_1, X_2, ..., X_n)$: Δ_2 are alpha expressions, then the following are alpha expressions.

   a) $p(X_1, X_2, ..., X_n)$: $\Delta_1 \vee \Delta_2$;

   b) $p(X_1, X_2, ..., X_n)$: $\Delta_1 \wedge \Delta_2$;

   c) $p(X_1, X_2, ..., X_n)$: $\Delta_1 \wedge \neg\Delta_2$.

$p(X_1, X_2, ..., X_n)$ is called the target point and the logical expression following the colon is called the qualification. The semantics of the alpha expression $p(X_1, X_2, ..., X_n)$: $\Gamma(p)$ is to construct a resource space RS consisting of axes X_1, X_2, ..., and X_n where for any point p if p satisfies $\Gamma(p)$ then p is a desirable point, otherwise p is a undesirable point.

The set ***AE*** is defined as the set of all alpha expressions, each of which can be used to represent a query in a certain resource space system.

Theorem 2 Any operation in the resource space algebra can be expressed by an alpha expression in the resource space calculus.

An algorithm similar to Codd's reduction algorithm has been constructed to transform resource space calculus to resource space algebra [2].

Theorem 3 Any alpha expression in the resource space calculus can be expressed by a series of operations in the resource space algebra.

4. Conclusion

The resource space algebra enables users or applications to directly and easily obtain the desired resources from the source resource spaces. The resource space calculus is a type of applied predicate calculus and provides a declarative style for describing the desired resources. The equivalence of the resource space algebra and the resource space calculus is discussed. They are the basis of the query language of the Resource Space Model [2-5].

References

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